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Supersonic Base Flow Problem in Presence of an Exhaust Jet

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For the axisymmetric base flow problem in presence of a jet, a systematic study is carried out taking into account all essential physical parameters. Using the method of characteristics for the inviscid outer flow and the method of Korst for the turbulent shear layers, the influence of the approaching boundary layer is introduced by the concepts of equivalent bleed and origin shift, and the ONERA angular criterion is used for the recompression process. A modification of the ONERA criterion for the two-stream recompression is proposed. The results are thoroughly discussed and compared with experiments, and estimations of the Reynolds number influence are presented.

I. Introduction

THE complexity of the flow pattern for the supersonic axisymmetric jet-afterbody interaction problem and uncertainties in the empirical parameters needed for calculations make the prediction of those flows difficult. In the past, studies based on the component approach of Korst et al.[1] overcame these difficulties by the introduction of an empirical recompression coefficient. These empirical coefficients for cylindrical[2] and conical[3] afterbodies depend upon the afterbody-nozzle geometry only and consequently indirectly account for many other physically essential factors. Hence, estimations of Reynolds number effects are impossible and the agreement between predictions and experiments may sometimes become unsatisfactory. The aim of the present study is to investigate the influence of all major components and parameters in order to establish an engineering type computational method and to compare the results obtained with well-established experimental results.

Following Ref. 1 the inviscid internal and external flow regions are treated with the method of characteristics. The mixing zones are assumed to be closely approximated by two-dimensional turbulent shear layers. The approaching boundary layer in the external flow is the most important influence factor. It is introduced by a combined application of the equivalent bleed concept[4,5] and the origin shift concept.[6-11]

This approach increases the significance of the spread rate parameter and emphasizes the current uncertainty in its experimental values. In order to study the corresponding effects, two different relations derived from experiments[12,13] and approximately bounding the experimental scatter region (see, e.g., Ref. 11) are used for this parameter. Also axisymmetric effects[14] and relaminarization effects at the jet boundary can be estimated by changing the spread rate parameter.

Two closure conditions for the separated flow region are available which are experimentally well-established and well-suited for an engineering approach, namely the recompression model of Page et al.[15] and the angular reattachment criterion of ONERA.[4,14] Both models give essentially similar results for thin boundary layers but the ONERA criterion has been preferred for the present study because of advantages in its application to the two-shear-layer confluence problem. Axisymmetric effects can be accounted for by proper corrections of the recompression parameters. Further, a modification of the ONERA criterion with regard to the two-stream recompression can be introduced.

Other possible inaccuracies (e.g., in the assumed velocity and temperature profiles of the shear layers, in the assumption of constant pressure mixing, or in the treatment of axisymmetric effects in the shear layers) are felt to be less important. Flow reversal in the base region may have significant influence but this is already implied in the recompression model.

Recent reviews concerning the component approach and the base pressure problem in general have been given by Page,[16] Tanner,[17] and Carrière et al.[18]

II. General Flow Model

The present study basically follows Ref. 1 in adapting the Chapman-Korst flow model for two-dimensional supersonic flow over a simple back step to the two-stream axisymmetric base pressure problem with turbulent shear layers.

Using trial values of the base pressure p_B, the inviscid external and internal flows are calculated by the method of characteristics for homentropic axisymmetric flows. The external flow upstream of the afterbody is assumed to be a parallel flow with constant Mach number. The internal flow may have any Mach number and deflection angle distribution at the nozzle exit. (For the present study it is assumed to be uniformly conical at the nozzle exit.) The flow conditions immediately downstream of the impingement point of the calculated inviscid boundaries result from the shock system at impingement, according to the requirement that static pressure and flow direction be equal (Fig. 1).

Two-dimensional shear layers are superimposed on the calculated inviscid boundaries of the separated flow region. The velocity profile in the mixing region is represented by the error function profile (other profiles may also be used successfully)

$$\phi = U/U_a = \tfrac{1}{2}[1+\mathrm{erf}(\eta)] \tag{1}$$

where U is the local velocity, U_a the velocity in the adjacent inviscid flow, and η the dimensionless coordinate:

$$\eta = \sigma y/x \tag{2}$$

and σ is the empirical spread rate parameter. The stagnation temperature profile for unit turbulent Prandtl number follows from Crocco's relationship

$$\Lambda = T_0/T_{0a} = \Lambda_B + (1-\Lambda_B)\phi \tag{3}$$

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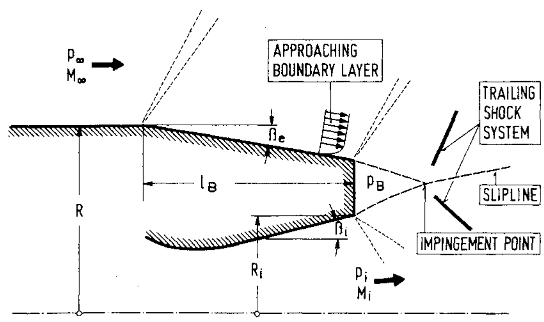

Fig. 1 Configuration and nomenclature.

where $\Lambda_B = T_B/T_{0a}$ (assuming zero velocity in the base region). This leads to the density ratio for nonisoenergetic mixing

$$\rho/\rho_a = (1-C_a^2)/(\Lambda - C_a^2\phi^2) \tag{4}$$

with C_a being the Crocco number of the adjacent inviscid flow.

Satisfying the integral momentum equation for the "restricted case" (i.e., without the initial boundary layer), the reference line of the velocity profile is localized with respect to the inviscid boundary. By application of the integral continuity equation, the position of the j streamline η_j will be found which separates the original mass flow of the adjacent stream from the mass entrained from the base region

$$\int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi \mathrm{d}\eta = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 \mathrm{d}\eta \tag{5}$$

where η_{Ra} and η_{Rb} refer to a nearly undisturbed streamline in the adjacent outer flow and to the stagnating base region, respectively.

At the impingement point the empirical recompression coefficient defines the fraction of the final pressure rise to which the shear layer has to be recompressed. This criterion determines the velocity ratio ϕ_d on the discriminating streamline η_d which is assumed to reattach by isentropic recompression up to stagnation. Therefore the mass flow rate

$$\sigma\bar{m} = \frac{\sigma m}{L\rho_a U_a} = \int_{\eta_d}^{\eta_j} \frac{\rho}{\rho_a}\phi \mathrm{d}\eta \tag{6}$$

leaves the base region within the corresponding shear layer with m being the physical mass flow and L the length of the layer. These mass fluxes of the external (m_e) and internal (m_i) shear layers have to be iteratively balanced to a possible bleed (m_b) into the base region in order to find the base pressure solution

$$m_e + m_i = m_b \tag{7}$$

As indicated by Eqs. (3) and (4), nonisoenergetic cases (different Λ_B and different ratio of specific heats γ) can be included which require in addition an energy balance equation for determining the temperature ratios Λ_{Be} and Λ_{Bi}.[1]

The studies presented will be restricted to isoenergetic cases ($\Lambda_B = 1$, $\gamma_e = \gamma_i$) because of the lack of other satisfactory experimental data, although all the following considerations can be extended to nonisoenergetic flows, different values of γ included.

Plume-induced separation on the afterbody surface can be treated by use of the Zukoski criterion[19] with empirical modifications for incipient separation effects.[20]

III. Influence of Initial Boundary Layers

The equivalent bleed concept[4,5] has been developed on the assumption that the shear layer velocity profiles are not strongly affected by disturbances induced as consequence of separation of thin approaching boundary layers. Hence, the momentum of a possible base bleed neglected, Eq. (5) is modified to

$$\int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi \mathrm{d}\eta = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 \mathrm{d}\eta - \sigma\frac{\theta}{L} \tag{8}$$

for a two-dimensional shear layer where θ is the momentum thickness of the initial boundary layer. Adding Eqs. (6) and (8) yields

$$\int_{\eta_d}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi \mathrm{d}\eta = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi^2 \mathrm{d}\eta - \sigma\left(\frac{\theta}{L} - \bar{m}\right) \tag{9}$$

Consequently, $\rho_a U_a \theta$ has to be treated as a base bleed m_b in Eq. (7) where for the axisymmetric case the different radii of impingement [left-hand side of Eq. (7)] and separation [right-hand side of Eq. (7)] must be considered with respect to mass flow continuity. In general this concept holds for both the boundary layer that developed on the afterbody and that in the nozzle. However, the nozzle boundary layer is always very thin because of the rapid expansions in the nozzle and at the nozzle exit, and its thickness is frequently unknown for experimental examples. Therefore, the internal boundary layer is generally neglected in the present calculations.

Regarding the finite thickness of the initial boundary layer, a virtual upstream displacement of the shear layer origin can be introduced which matches the fictitious shear layer at the separation point to the actual boundary-layer parameters immediately downstream of separation. This origin shift concept has been proposed by Kirk[6] and developed by different authors.[7-11] Matching the entire momentum and mass flow of the shear layer to the corresponding boundary-layer data at separation, the following relation is obtained

$$\frac{\sigma\theta}{x_0} = \int_{\eta_{Rb}}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi(1-\phi)\mathrm{d}\eta = \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a}\phi \mathrm{d}\eta \tag{10}$$

where x_0 denotes the origin shift distance. The last form of Eq. (10) is derived by using the definition of the j streamline [Eq. (5)] for the restricted case and it indicates that the equivalent bleed mass flow is represented by the whole mass flow below the j streamline at separation. This concept includes in its general deduction[8] the equivalent bleed concept (where in Eqs. (8) and (9) only the length of the mixing layer L has to be replaced by its effective value) and a lateral origin shift of the shear layer.

A different approach to the virtual origin displacement is given in Ref. 10 and is shown to be experimentally well-confirmed. This approach is based on the relation

$$\frac{\sigma\theta}{x_0} = \int_{\eta_j}^{\eta_{Ra}} \frac{\rho}{\rho_a}\phi(1-\phi)\mathrm{d}\eta = \int_{\eta_{Rb}}^{\eta_j} \frac{\rho}{\rho_a}\phi^2 \mathrm{d}\eta \tag{11}$$

where again the last form arises from application of Eq. (5). Equation (11) matches the momentum thickness of the shear layer above the j streamline to the boundary-layer momentum thickness at separation and implies the lack of momentum in the boundary-layer flow compared with parallel flow to be equal to the momentum of the shear flow below the j streamline.

Other origin shift procedures are possible, e.g., accounting for the boundary-layer shape parameter.[11] However, only Eq. (10) and (11) are applied in the present calculations and

Eq. (11) is used if no special indication is made. Lateral origin shifts are ignored, because great complications would arise for the axisymmetric calculations.

The boundary-layer momentum thickness used in Eqs. (8-11) has been tacitly assumed to have the value after the rapid expansion (or compression) at separation. It is related to the approaching boundary-layer data using the simple method of Nash[7]

$$\frac{\theta}{\theta_0}=\frac{\rho_{0a}U_{0a}M_{0a}^2}{\rho_a U_a M_a^2} \tag{12}$$

denoting the approaching flow before separation by the subscript 0 and M being the Mach number. Although many other treatments are known (see, e.g., Refs. 17 and 21), Eq. (12) has been used since the differences from other methods can be shown to have a negligibly small influence upon the base pressure calculations.

According to Eqs. (8-12) an estimation of Reynolds number effects is possible if the behavior of boundary-layer momentum thickness is known as a function of Reynolds number and Mach number. Since the momentum thickness is related directly to the integral wall friction coefficient, the necessary extrapolations are fitted to the incompressible wall friction law and the usual representation of the compressibility effects (see, e.g., Refs. 22 and 23).

IV. Spread Rate Parameter

When calculating base pressures with the aid of empirical recompression coefficients, the solutions are little affected by the use of different laws for the spread rate parameter σ. The pressure behind a simple backstep is independent of σ while the two-stream recompression problem may show a very small influence because of different Mach numbers in the adjacent internal and external streams. If the approach boundary layer is introduced, the values of σ attain great importance since they occur as factors in all equivalent bleed and origin shift formulas.

Unfortunately, the experimental values for σ exhibit tremendous scatter.[11,17,18] Hence, to study this influence two σ relations have been chosen for the present calculations, namely the relations of Korst and Tripp[12] and Channapragada,[13] which represent approximately the lower and upper bounds of the experimental scatter respectively. Korst and Tripp suggested the equation

$$\sigma=12+2.758M_a \tag{13}$$

and Channapragada proposed

$$\sigma=12/\{R[1+(1-C_a^2)/\Lambda_b]\} \tag{14}$$

where R is presently approximated by

$$R=0.5(C_a^2-0.7)^2+0.25 \quad \text{for } C_a^2<0.7$$

$$R=0.25 \quad \text{for } C_a^2\geq 0.7$$

The higher σ values of Eq. (14) emphasize the boundary-layer influence in comparison to the lower values of Eq. (13). In the calculated examples Eq. (13) is used if not indicated otherwise.

According to Ref. 14 axisymmetric effects in the shear layer can be taken into account by changing the spread parameter corresponding to the similarity law

$$\sigma_{\text{ax}}=\sigma_{\text{plane}}/F \tag{15a}$$

with

$$F=\frac{1}{Lr_R}\int_0^L r\,\mathrm{d}s \tag{15b}$$

where r_R is the reattachment radius and ds a differential of the mixing length L. In all calculations, even for small nozzle radii, Eq. (15a) produces only negligible effects compared to those introduced by changing from Eq. (13) to Eq. (14). Consequently, numerical results for applications of Eq. (15a) are not given in the present paper but with the aid of the parameter F [Eq. (15b)] corrections for the axisymmetric recompression process[14] are introduced (see Sec. V).

It has been frequently observed in experiments that relaminarization effects take place if a turbulent boundary layer undergoes a rapid expansion after separation. These effects can be accounted for in principal[24] using the component approach. However, in the present study only the maximum possible contribution of such effects is estimated because the method of Ref. 24 greatly complicates the computation of the two-stream base pressure problem. For this purpose some calculations have been carried out by setting σ to a very high value at the internal boundary (i.e., neglecting totally the entrainment there) where such effects might become essential for high ratios of jet pressure to external pressure.

V. Recompression Models

Many efforts have been undertaken to model the recompression process properly in the component approach (see Refs. 9, 11, 16-18). From an engineering point of view the models of Page et al.[15] and the ONERA angular criterion[4] are of outstanding interest because both give simple correlations and contain comprehensive empirical information. Both methods use the assumptions that the reattachment pressure p_R approximately equals the stagnation pressure on the discriminating streamline p_{0d} and that the recompression depends essentially upon the angular deflection during this process.

Attempts to implement the Page model into the present computer program caused serious convergence problems due to the iterative procedure, necessary to calculate the base pressure,[9] being complicated for the two-stream problem by the requirement of Eq. (7). Therefore no results are presented with this method but the similarity of both models is shown for thin boundary layers (see below).

The ONERA criterion is based on two experimentally evident facts. First, the recompression deflection angle for the limiting case of no bleed and no initial boundary layer $\bar{\psi}$ cannot be predicted theoretically but must be taken from experiments. Second, the changes of the actual recompression deflection angle ψ resulting from the base bleed or boundary-layer influence are correctly predicted by the theory of Korst (using equivalent bleed and origin shift concepts in case of boundary-layer influence).

Hence, the ONERA criterion yields for thin boundary layers and small bleed

$$\psi=\bar{\psi}(M_a)+C_q\left.\frac{\partial\psi}{\partial C_q}\right|_{\eta=\eta_j} \tag{16}$$

where C_q represents a general bleed coefficient (present definition different from Refs. 4, 14, and 18)

$$C_q=-\sigma\left(\frac{m_b}{\rho_a U_a L}+\frac{\theta}{L}\right) \tag{17}$$

At present Eq. (16) holds for isoenergetic case only ($\Lambda_B=1$, $\gamma_e=\gamma_i$) because of the lack of sufficient nonisoenergetic experimental data. The empirical curve for $\bar{\psi}$ is given in Fig. 2 and can be approximated by[14]

$$\bar{\psi}\simeq 57.3(0.569\text{-}0.5096/M_a) \tag{18}$$

where $\bar{\psi}$ is the degrees. According to Fig. 2 this approximation is well-established by experiments within the range $2\leq M_a\leq 4$. The comparison with the corresponding limiting angle

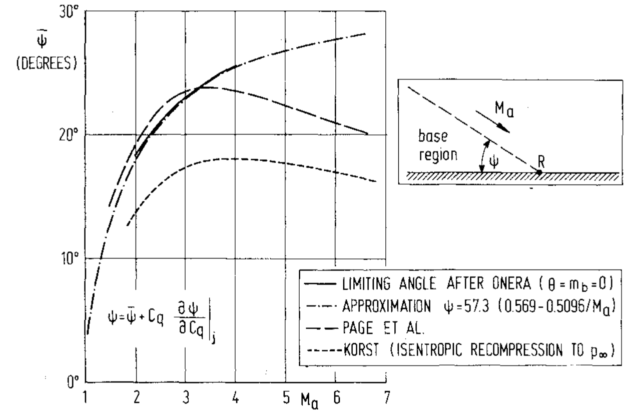

Fig. 2 Limiting reattachment deflection angle $\bar{\psi}$ (two-dimensional).

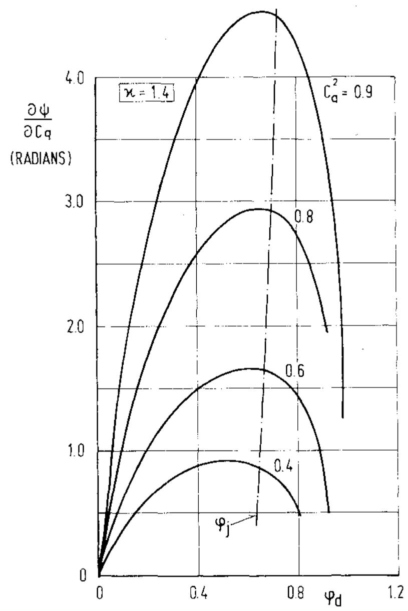

Fig. 3 Gradient of reattachment deflection angle ψ against discriminating streamline velocity ratio ϕ_d.

resulting from the Page model shows close agreement over a broad range, while the values of the original Korst theory (assuming isentropic recompression up to the pressure downstream of the shock system) differ considerably. The gradient of ψ in Eq. (16) can be derived theoretically from the relations of Secs. II and III, the condition of isentropic compression on the discriminating streamline, and the Prandtl-Meyer function (which is valid as a first approximation for the adjacent external flow, since the shock is formed far outside)

$$\left.\frac{\partial \psi}{\partial C_q}\right|_j = \frac{1}{\sqrt{\pi}} e^{-\eta_j^2} \frac{\sqrt{M_R^2 - 1}}{1 - \phi_j^2} \tag{19}$$

with the adjacent flow Mach number at the reattachment point[18]

$$M_R^2 = M_a^2 (1 - \phi_j^2) \tag{20}$$

Figure 3 shows the gradient $\partial\psi/\partial C_q$ as a function of C_q and the curves justify that the $\partial\psi/\partial C_q$ value at the j streamline can be used over a quite broad C_q range with acceptable accuracy because of the maximums close to the ϕ_j position. Moreover, in Fig. 4 this gradient at ϕ_j is compared to the corresponding one which can be computed from the Page criterion for thin boundary layers (without accounting for reduced shear layer

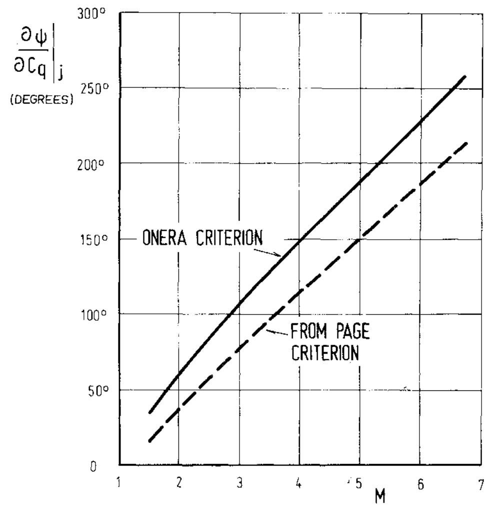

Fig. 4 Comparison of different reattachment angle gradients.

length in the Page criterion). Despite quantitative deviations both curves are similar in essence. Since the limiting angles $\bar{\psi}$ agree well (Fig. 2), a similar solution behavior can be expected.

Account can also be given to axisymmetric effects in the ONERA criterion by use of the universal axisymmetric reattachment correction[14] which can be approximated by

$$\Delta\bar{\psi} = \bar{\psi}_{\text{ax}} - \bar{\psi}_{\text{plane}} = 2.7\text{-}3.5\tanh(5.4F - 4.4) \tag{21}$$

with all angles in degrees and the parameter F being defined by Eq. (15b). Only small effects result from this correction in the calculated examples, even if the nozzle radius is small compared to the external base radius. A numerical example is given below.

In the form given above, the ONERA angular criterion has already been applied to two-stream axisymmetric reattachment processes with remarkable success.[25] But in all these cases the procedure for a simple backstep has been directly transferred to the two-stream reattachment by identifying the wall direction with the flow direction downstream of the impingement shock system. Consequently, the interference effects between the recompression zones of both streams are totally neglected. Since in the present calculations a worse agreement with experiments occured at high ratios of jet pressure to external pressure, a first approximation for such interferences has been introduced.

According to the systematic experiments at ONERA,[26] conditions downstream of reattachment can affect the recompression process up to the reattachment point only in a very restricted manner. Moreover, the unmodified application of the ONERA criterion results in different reattachment pressures for the internal and external recompression zones. Since this is not physically plausible, the reattachment criterion has been modified for the present calculations in order to give equal reattachment pressures p_R for both zones by determining iteratively the associated effective reattachment direction (corresponding to wall direction in case of a simple back step). This concept is only a first approach in order to account for this type of interference since other effects are neglected, e.g., slipline curvature (which results immediately from the different flow directions at reattachment and downstream of the impingement region and is comparable to the reattachment to a curved wall), different lengths of the recompression zones, and the wall friction being suppressed (compared with the case of a simple backstep).

The recompression pressure rise up to the reattachment point is given by isentropic flow on the discriminating streamline

$$\frac{p_{0d}}{p_B} = \left[\frac{1+M_a^2(\gamma-1)/2}{1+M_a^2(1-\phi_d^2)(\gamma-1)/2}\right]^{\frac{\gamma}{\gamma-1}} \tag{22}$$

with γ being the ratio of the specific heats and M_R introduced after Eq. (20). Since the usual applications of the ONERA criterion the deflection angle ψ is known after computation of the inviscid boundaries, the general bleed coefficient can be immediately calculated from Eq. (16) and then the terms on the left-hand side of Eq. (7) are known.

Now, only the sum of both values for ψ is known and, in addition, the values of p_{0d}/p_B after Eq. (22) have to be equal. Therefore, it is useful for the numerical procedure to derive a simple relation between ϕ_d and C_q comparable to Eq. (16). This is done by the first terms of the series expansion

$$\phi_d = \phi_j + C_q \frac{\partial \phi_d}{\partial C_q}\bigg|_j + \frac{C_q^2}{2}\frac{\partial^2 \phi_d}{\partial C_q^2}\bigg|_j$$

$$= \phi_j + C_q \frac{\partial \phi_d}{\partial C_q}\bigg|_j \left\{1 - \frac{C_q}{\phi_j^2(1-C_a^2)}\left[(1+\phi_j^2 C_a^2)\frac{e^{-\eta_j^2}}{2\sqrt{\pi}}\right.\right.$$

$$\left.\left. + (1-\phi_j^2 C_a^2)\eta_j \phi_j\right]\right\} \tag{23a}$$

where

$$\frac{\partial \phi_d}{\partial C_q}\bigg|_j = \frac{1-\phi_j^2 C_a^2}{1-C_a^2}\,\frac{1}{\phi_j}\,\frac{e^{-\eta_j^2}}{\sqrt{\pi}} \tag{23b}$$

The second-order term has been included since the gradient $\partial\phi_d/\partial C_q$ shows a strong variation within the whole ϕ_d range in contrast to the behavior of $\partial\psi/\partial C_q$ in Eq. (16). The procedure suggested above is called the "modified ONERA angular criterion" in the following section.

VI. Results

In comparing the flow model with experiments, a central problem arises due to the restriction of the origin shift concept to thin boundary layers of about $\theta/L \leq 0.01$.[25] Since the length of the mixing layer L is usually shorter than the body radius R for jet-afterbody interference problems, this results in the limitation of at most $\theta/R \leq 0.01$. Consequently, only such experiments which do not exceed this restriction too far can be practically used. Furthermore many experiments are lacking boundary-layer information in general. Therefore, only a few experiments with sufficient boundary-layer data are compared with the theory in the present paper. The momentum thickness restriction mentioned above is closely fulfilled by only one of these experimental series.[27]

The calculations shown in Fig. 5 are carried out for different shear-layer/recompression models at an external Mach number of $M_\infty = 2$ for a cylindrical afterbody. They are compared to experiments[28] over a considerable range of the ratio of static pressure at the nozzle exit p_i to static pressure p_∞ in the external flow in front of the afterbody. For curve 1 the original ONERA criterion the origin shift after Eq. (11) and the spread rate parameter after Eq. (13) are used. The base pressure is somewhat overpredicted but shows no larger deviations than the calculation with the empirical recompression coefficient after Addy (curve 8) which underpredicts p_B and fails for $p_i/p_\infty > 12$. (This failure is due to the fact that a completely oblique shock system cannot exist at impingement if the base pressure falls below a definite value at given pressure ratio p_i/p_∞. The real shock pattern must become similar to the "Mach reflection" at a solid wall.)

Changing to the σ law [Eq. (14)] raises the base pressure uniformly in the whole range (curve 2), as would be expected since the shear layers are less capable of entraining fluid at higher values of σ. The use of the origin shift after Hill [curve

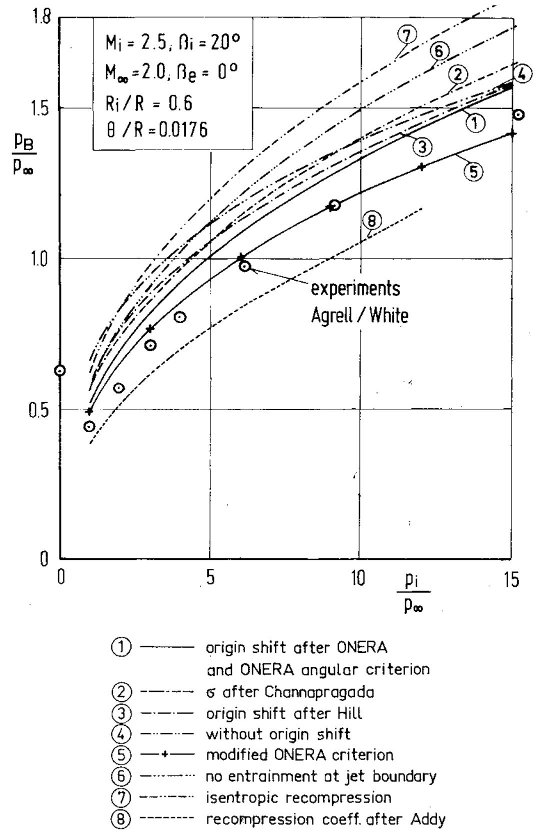

Fig. 5 Jet-Cylindrical afterbody configuration, p_B results compared with Ref. 28.

3, Eq. (10)] also elevates the base pressure level because the predicted shift x_0 is smaller than that of Eq. (11). But these pressure rises are in general small compared with the case of completely neglecting the origin shift concept (curve 4). Only at high pressure ratios p_i/p_∞ does the effect of origin shift diminish significantly.

Remarkably improved agreement with the experiments results if the modified version of the ONERA criterion is used (curve 5). The base pressure is well-estimated up to high pressure ratios where some amount of underprediction takes place. A number of reasons are possible for the underprediction at the higher pressure ratios. First, relaminarization of the internal boundary as mentioned in Sec. IV is more pronounced at high pressure ratios and raises the base pressure, as can be seen from curve 6 where the entrainment of the jet boundary is almost totally neglected (compared to curve 1). Second, it is known[20] that submerged normal shocks have been observed at high pressure ratios in the impingement regions, instead of the system of purely oblique shocks supposed in the calculations. It is also possible that the other interference effects discussed in Sec. V contribute to the deviations noted.

Figure 6 shows the estimation of Reynolds number effects for a single pressure ratio among the cases of Fig. 5. The boundary-layer momentum thicknesses are predicted by the method indicated in Sec. III. Curve 1 is now calculated with the aid of the modified ONERA criterion. A steep gradient of this curve occurs only in Reynolds number regions which are generally of no practical interest. In the usual Reynolds number range, only small to moderate changes of base pressure are estimated.

Passing over to the original ONERA criterion (curve 2), to another σ law (curve 3), or accounting for both these effects (curve 4), the agreement with the single experimental point decreases quantitatively. But qualitative disagreement can be observed only if the origin shift is omitted (curves 5 and 6). Therefore from Fig. 6 the conclusion can be drawn that Reynolds number effects can be expected to remain usually

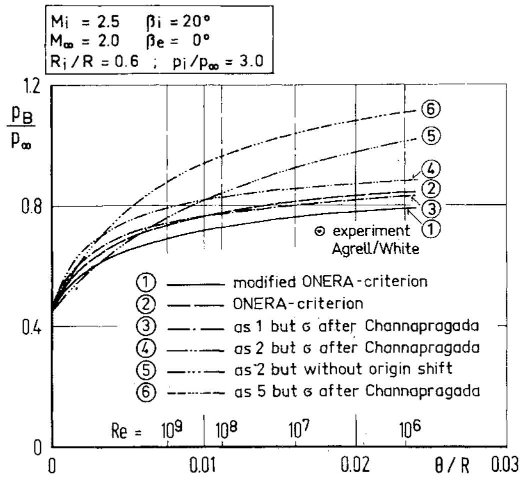

Fig. 6 Estimated Reynolds number influence for jet-cylindrical afterbody configuration.

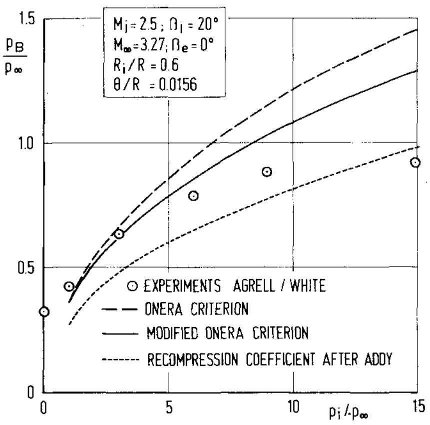

Fig. 7 Jet-cylindrical afterbody configuration, p_B results compared with Ref. 28.

small and that origin shift procedures are essentially necessary to predict realistic base pressures.

In Fig. 7 calculated results for a case with a higher external Mach number ($M_\infty = 3.27$) are presented. A considerable overprediction of base pressure appears for high pressure ratios. Besides the reasons mentioned above, this may be attributed to lip shock effects which can be computed only by applying rotational methods of characteristics and have already been supposed to lead to similar deviations in the two-dimensional case.[9] In Fig. 8 results for a configuration with a conical afterbody are given having a boattail angle of 6 deg. Again considerable agreement with experiments[28] is obtained using the modified ONERA criterion.

In the case of Fig. 8 no separation occurs on the boattail surface. If cases with flow separation on the afterbody are treated with the present method, in general the agreement between predicted and measured separation points is reduced in comparison with calculations using empirical recompression coefficients.[20] Further work is needed concerning this problem.

Comparison between another set of experiments[27] and calculations is made in Fig. 9 for cylindrical afterbodies. The base drag coefficient C_{DB} is plotted against the pressure ratio p_i/p_∞ for different values of the ratio of nozzle radius R_i to body radius R. Satisfactory agreement is obtained, especially for large nozzle radius. But it should be kept in mind that the results for C_{DB} also depend on the radius ratio. Therefore the deviations of p_B/p_∞ are more uniform for different values of R_i/R. Axisymmetric reattachment corrections do not play an important role, as can be seen from the case of $R_i/R = 0.2$ where a result calculated by neglecting Eq. (21) is presented. In Fig. 10 further comparisons with the experiments of Ref. 27 are shown and the agreement with theory is satisfactory (stagnation pressure ratio on the horizontal axis instead of static pressure ratio in the other figures, this ratio being identical with p_i/p_∞ for $M_i = 2$ because of equal Mach numbers in both streams). However, although the experiments of Ref. 27 exhibit the relatively thinnest boundary layers found in the literature and are therefore very reliable for comparisons with the theory, they unfortunately extend only over a small range of pressure ratios p_i/p_∞.

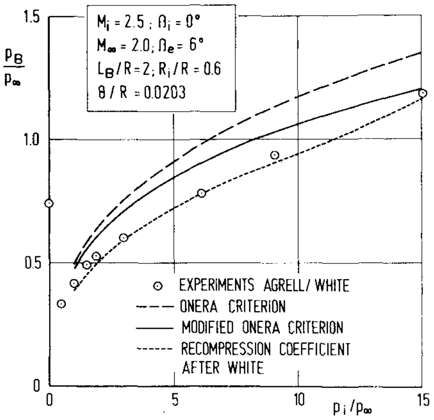

Fig. 8 Jet-conical afterbody configuration, p_B results compared with Ref. 28.

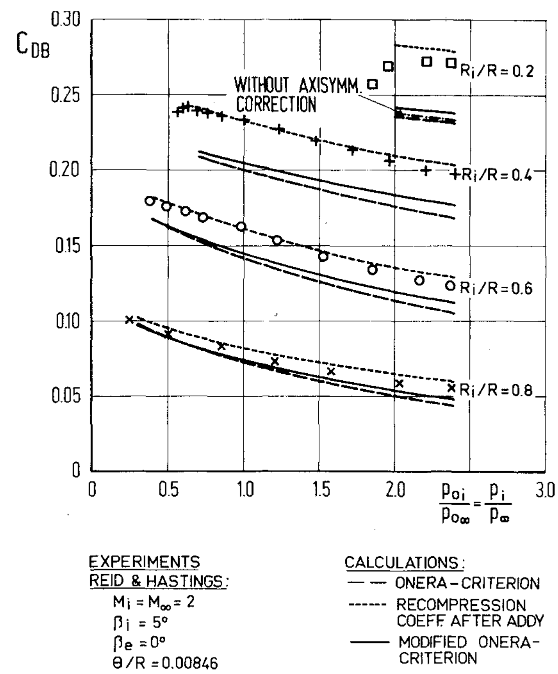

Fig. 9 Jet-cylindical afterbody configuration, variation of nozzle to body diameter ratio, compared with Ref. 27.

Finally, in Fig. 11 experimental results for a configuration tested at ONERA[25] are compared with the theory. (The complete data set of this configuration has been received by private communication.) The theory predicts the correct trend but the differences with the experiments are larger than for corresponding theoretical results calculated at ONERA.[25] Obviously, in Ref. 25 the application of the original ONERA criterion includes bleed momentum terms[29] which additionally account for the mutual interaction between both shear layers. But no detailed comparison with this computational method can be presented since the ONERA procedure is not yet known in all details.

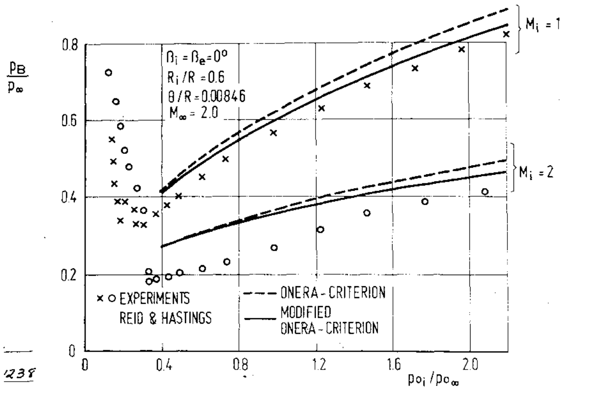

Fig. 10 Jet-cylindrical afterbody configuration, variation of jet Mach number, compared with Ref. 27.

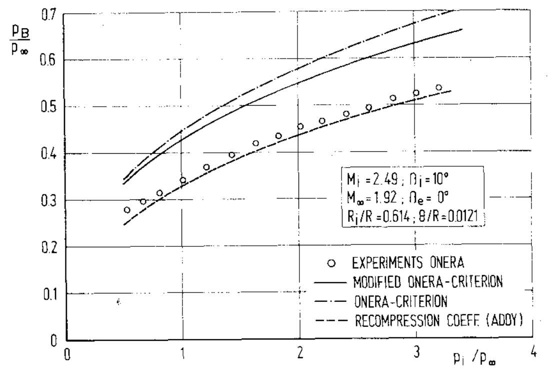

Fig. 11 Jet-cylindrical afterbody configuration, p_B results compared with Ref. 25.

VII. Conclusions

Accounting for all physically relevant influence parameters, a satisfactory prediction of base pressures up to moderate ratios of jet pressure to external pressure is achieved for the jet-afterbody interference problem in supersonic flow if flow separation on the boattail surface is excluded. With regard to the two-stream recompression process, modifications of the usual recompression criteria appear to be necessary and are included in a first approximation form. Some reasons for the deviations observed at high pressure ratios are evident. Further work concerning this special problem and the flow separation on the afterbody surface is needed. Additional experiments exhibiting thin approaching boundary layers are of high interest.

Estimations of Reynolds number effects on the base pressure show small variations within the usual Reynolds number range. Large differences occur only at very high Reynolds numbers which are generally not of practical interest. Therefore, the resulting deviations between model experiments and free flight can be expected to be generally small.

Further details of the present study can be found in Ref. 30.

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